

**Amendments to the Claims:**

This listing of claims will replace all prior versions and listings of claims in the application:

**Listing of Claims:**

1-12 (Cancelled)

13. (New) A system for producing asymmetric cryptographic keys, said keys comprising  $m \geq 1$  private values  $Q_1, Q_2, \dots, Q_m$  and  $m$  respective public values  $G_1, G_2, \dots, G_m$ , the system comprising:

a processor; and

a memory unit coupled to the processor, the memory unit storing a set of instructions, which when executed cause the processor to execute the following acts:

selecting a security parameter  $k$ , wherein  $k$  is an integer greater than 1;

selecting  $m$  base numbers  $g_1, g_2, \dots, g_m$ , wherein each base number  $g_i$  (for  $i = 1, \dots, m$ ) is an integer greater than 1;

determining a modulus  $n$ , wherein  $n$  is a public integer equal to the product of at least two prime factors  $p_1, \dots, p_f$ , at least two of these prime factors, say  $p_1$  and  $p_2$ , being such that  $p_1 \equiv 3 \pmod{4}$ ,  $p_2 \equiv 3 \pmod{4}$ , and such that  $p_2$  is complementary to  $p_1$  with respect to one of the base numbers;

calculating the public values  $G_i$  for  $i = 1, \dots, m$  through  $G_i \equiv g_i^2 \pmod{n}$ ; and

calculating the private values  $Q_i$  for  $i = 1, \dots, m$  by solving either the equation  $G_i \cdot Q_i^v \equiv 1 \pmod{n}$  or the equation  $G_i \equiv Q_i^v \pmod{n}$ , wherein the public exponent  $v$  is such that  $v = 2^k$ .

14. (New) The system according to claim 13, wherein the number  $(f - e)$  (where  $e \geq 0$ ) of prime factors of the modulus  $n$  which are congruent to 3 mod 4 is larger than 2, and those prime factors  $p_{j+1}$  for  $2 \leq j \leq m$  which are congruent to 3 mod 4 are determined iteratively as follows:

the profile <sub>$j$</sub> ( $g_j$ ) of  $g_j$  with respect to the prime factors  $p_1, p_2, \dots, p_j$  is computed, and

if profile <sub>$j$</sub> ( $g_j$ ) is flat, then the prime factor  $p_{j+1}$  is chosen such that  $p_{j+1}$  is complementary to  $p_1$  with respect to  $g_j$ ; else, a number  $g$  is chosen among the  $(j - 1)$  base numbers  $g_1, g_2, \dots, g_{j-1}$  and all of their multiplicative combinations, such that profile <sub>$j$</sub> ( $g$ ) = profile <sub>$j$</sub> ( $g_j$ ), then  $p_{j+1}$  is chosen such that profile <sub>$j+1$</sub> ( $g_j$ )  $\neq$  profile <sub>$j+1$</sub> ( $g$ ),

wherein the last prime factor  $p_{f-e}$  congruent to 3 mod 4 is, in the case that  $f - e \leq m$ , chosen such that  $p_{f-e}$  is complementary to  $p_1$  with respect to all of the base numbers  $g_i$  such that  $f - e \leq i \leq m$  and whose profile profile <sub>$f-e-1$</sub> ( $g_i$ ) is flat.

15. (New) The system according to claim 13, wherein the number  $e$  of prime factors of the modulus  $n$  which are congruent to 1 mod 4 is at least equal to 1, and each such prime factor is determined as follows:

a candidate prime number  $p$  is chosen, such that the Legendre symbol of each base number  $g_i$  (for  $i = 1, \dots, m$ ) with respect to  $p$  is equal to +1,

the integer  $t$  is computed such that  $(p - 1)$  is divisible by  $2^t$ , but not by  $2^{t+1}$ ,

the integer  $s = (p - 1 + 2^t) / 2^{t+1}$  is computed,

an integer  $b \equiv h^{p-1/2^t} \bmod p$ , where  $h$  is a non-quadratic residue of the body of integers modulo  $p$ , is computed,

the  $m$  integers  $r_i \equiv g_i^{2^s} \bmod p$  for  $i = 1, \dots, m$  are computed,

an integer  $u$  is initialized to  $u = 0$ ,

the following sequence of steps, where  $i$  is initialized to 1, is iteratively implemented:

an integer  $w$  is initialized to  $w = r_i$ ,

if  $r_i = \pm g_i$ , the value of  $i$  is incremented and a sequence of steps with the new value of  $i$  is proceeded to if  $i < m$ , whereas the candidate prime number  $p$  is accepted as a factor of the modulus  $n$  if  $i = m$ ,

if  $r_i \neq \pm g_i$ :

an integer  $jj$  is initialized to 1,

the following sequence of steps, where an integer  $ii$  is initialized to 1, is iteratively implemented:

$x \equiv w^2 / g_i^2 \bmod p$  is computed,

$y \equiv x^{2^{t-ii-1}} \bmod p$  is computed, and

if  $y = +1$ , the sequence is terminated at the current value of  $ii$ ,

if  $y = -1$ ,  $jj$  is assigned the value  $jj = 2^{ii}$ , the number  $w$  is assigned a new value equal to the old value multiplied by  $b^{jj}$  modulo  $p$ , and

for  $ii < t - 2$ , the value of  $ii$  is incremented and a new iteration is proceeded to with the new value of  $ii$ ,

for  $ii = t - 2$ , the value of number  $u$  is updated through the relation  $jj = 2^{t-u}$ , and

if  $t - u < k$ , the candidate prime number  $p$  is rejected as a factor of the modulus  $n$ ,

if  $t - u > k$ , the value of  $i$  is incremented and a sequence of steps with the new value of  $i$  is proceeded to if  $i < m$ , whereas the candidate prime number  $p$  is accepted as a factor of the modulus  $n$  if  $i = m$ .

16. (New) The system according to claim 13, wherein, to compute the  $f \cdot m$  private components  $Q_{i,j}$  of the private values  $Q_1, Q_2, \dots, Q_m$ , the following steps are implemented for each couple  $(i, j)$ :

an integer  $t$  is determined, which is equal to 1 if  $p_j$  is congruent to 3 mod 4, and to the value obtained for  $t$  according to claim 15 if  $p_j$  is congruent to 1 mod 4,

an integer  $u$  is determined, which is equal to 0 if  $p_j$  is congruent to 3 mod 4, and to the value obtained for  $u$  according to claim 15 if  $p_j$  is congruent to 1 mod 4,

the integer  $z \equiv G_i^s \text{ mod } p_j$  is computed, where  $s = (p - 1 + 2^t) / 2^{t+1}$ ,

all the numbers  $zz$  are being considered, which:

if  $u = 0$ , are such that  $zz = z$  or such that  $zz$  is equal to the product modulo  $p_j$  of  $z$  by each of the  $2^{ii-1}$   $2^{ii}$ -th primitive roots of unity, for  $ii$  ranging from 1 to  $\min(k, t)$ ,

if  $u > 0$ , are such that  $zz$  is equal to the product modulo  $p_j$  of  $za$  by each of the  $2^k$   $2^k$ -th roots of unity, where  $za$  is the value obtained for  $w$  according to claim 15, and

for each such number  $zz$ , a value for the component  $Q_{i,j}$  is obtained by taking  $Q_{i,j}$  equal to  $zz$  if the equation  $G_i \equiv Q_i^v \pmod{n}$  is used, or to the inverse of  $zz$  modulo  $p_j$  if  $G_i \cdot Q_i^v \equiv 1 \pmod{n}$  is used for this value of  $i$ .

17. (New) A computer-readable storage medium storing instructions for producing asymmetric cryptographic keys, said keys comprising  $m \geq 1$  private values  $Q_1, Q_2, \dots, Q_m$  and  $m$  respective public values  $G_1, G_2, \dots, G_m$ , the medium storing instructions, which when executed cause a processor to carry out the following acts:

selecting a security parameter  $k$ , wherein  $k$  is an integer greater than 1;

selecting  $m$  base numbers  $g_1, g_2, \dots, g_m$ , wherein each base number  $g_i$  (for  $i = 1, \dots, m$ ) is an integer greater than 1;

determining a modulus  $n$ , wherein  $n$  is a public integer equal to the product of at least two prime factors  $p_1, \dots, p_f$ , at least two of these prime factors, say  $p_1$  and  $p_2$ , being such that  $p_1 \equiv 3 \pmod{4}$ ,  $p_2 \equiv 3 \pmod{4}$ , and such that  $p_2$  is complementary to  $p_1$  with respect to one of the base numbers;

calculating the public values  $G_i$  for  $i = 1, \dots, m$  through  $G_i \equiv g_i^2 \pmod{n}$ ; and

calculating the private values  $Q_i$  for  $i = 1, \dots, m$  by solving either the equation  $G_i \cdot Q_i^v \equiv 1 \pmod{n}$  or the equation  $G_i \equiv Q_i^v \pmod{n}$ , wherein the public exponent  $v$  is such that  $v = 2^k$ .

18. (New) The computer-readable storage medium storing instructions according to claim 17, wherein the number  $(f - e)$  (where  $e \geq 0$ ) of prime factors of the modulus  $n$  which are congruent to 3 mod 4 is larger than 2, and those prime factors  $p_{j+1}$  for  $2 \leq j \leq m$  which are congruent to 3 mod 4 are determined iteratively as follows:

the profile  $\text{profile}_j(g_j)$  of  $g_j$  with respect to the prime factors  $p_1, p_2, \dots, p_j$  is computed, and

if  $\text{profile}_j(g_j)$  is flat, then the prime factor  $p_{j+1}$  is chosen such that  $p_{j+1}$  is complementary to  $p_1$  with respect to  $g_j$ ; else, a number  $g$  is chosen among the  $(j - 1)$  base numbers  $g_1, g_2, \dots, g_{j-1}$  and all of their multiplicative combinations, such that  $\text{profile}_j(g) = \text{profile}_j(g_j)$ , then  $p_{j+1}$  is chosen such that  $\text{profile}_{j+1}(g_j) \neq \text{profile}_{j+1}(g)$ ,

wherein the last prime factor  $p_{f-e}$  congruent to 3 mod 4 is, in the case that  $f - e \leq m$ , chosen such that  $p_{f-e}$  is complementary to  $p_1$  with respect to all of the base numbers  $g_i$  such that  $f - e \leq i \leq m$  and whose profile  $\text{profile}_{f-e-1}(g_i)$  is flat.

19. (New) The computer-readable storage medium storing instructions according to claim 17, wherein the number  $e$  of prime factors of the modulus  $n$  which are congruent to 1 mod 4 is at least equal to 1, and each such prime factor is determined as follows:

a candidate prime number  $p$  is chosen, such that the Legendre symbol of each base number  $g_i$  (for  $i = 1, \dots, m$ ) with respect to  $p$  is equal to  $+1$ ,

the integer  $t$  is computed such that  $(p-1)$  is divisible by  $2^t$ , but not by  $2^{t+1}$ ,

the integer  $s = (p-1+2^t)/2^{t+1}$  is computed,

an integer  $b \equiv h^{p-1/2^t} \bmod p$ , where  $h$  is a non-quadratic residue of the body of integers modulo  $p$ , is computed,

the  $m$  integers  $r_i \equiv g_i^{2^s} \bmod p$  for  $i = 1, \dots, m$  are computed,

an integer  $u$  is initialized to  $u = 0$ ,

the following sequence of steps, where  $i$  is initialized to 1, is iteratively implemented:

an integer  $w$  is initialized to  $w = r_i$ ,

if  $r_i = \pm g_i$ , the value of  $i$  is incremented and a sequence of steps with the new value of  $i$  is proceeded to if  $i < m$ , whereas the candidate prime number  $p$  is accepted as a factor of the modulus  $n$  if  $i = m$ ,

if  $r_i \neq \pm g_i$ :

an integer  $jj$  is initialized to 1,

the following sequence of steps, where an integer  $ii$  is initialized to 1, is iteratively implemented:

$x \equiv w^2 / g_i^2 \bmod p$  is computed,

$y \equiv x^{2^{t-ii-1}} \bmod p$  is computed, and

if  $y = +1$ , the sequence is terminated at the current value of  $ii$ ,

if  $y = -1$ ,  $jj$  is assigned the value  $jj = 2^{ii}$ , the number  $w$  is assigned a new value equal to the old value multiplied by  $b^{jj}$  modulo  $p$ , and

for  $ii < t - 2$ , the value of  $ii$  is incremented and a new iteration is proceeded to with the new value of  $ii$ ,

for  $ii = t - 2$ , the value of number  $u$  is updated through the relation  $jj = 2^{t-u}$ , and

if  $t - u < k$ , the candidate prime number  $p$  is rejected as a factor of the modulus  $n$ ,

if  $t - u > k$ , the value of  $i$  is incremented and a sequence of steps with the new value of  $i$  is proceeded to if  $i < m$ , whereas the candidate prime number  $p$  is accepted as a factor of the modulus  $n$  if  $i = m$ .

20. (New) The computer-readable storage medium storing instructions according to claim 17, wherein, to compute the  $f \cdot m$  private components  $Q_{i,j}$  of the private values  $Q_1, Q_2, \dots, Q_m$ , the following steps are implemented for each couple  $(i, j)$ :

an integer  $t$  is determined, which is equal to 1 if  $p_j$  is congruent to 3 mod 4, and to the value obtained for  $t$  according to claim 15 if  $p_j$  is congruent to 1 mod 4,

an integer  $u$  is determined, which is equal to 0 if  $p_j$  is congruent to 3 mod 4, and to the value obtained for  $u$  according to claim 15 if  $p_j$  is congruent to 1 mod 4,



the integer  $z \equiv G_i^s \bmod p_j$  is computed, where  $s = (p - 1 + 2') / 2'^{+1}$ ,

all the numbers  $zz$  are being considered, which:

if  $u = 0$ , are such that  $zz = z$  or such that  $zz$  is equal to the product modulo  $p_j$  of  $z$  by each of the  $2^{ii-1}$   $2^{ii}$ -th primitive roots of unity, for  $ii$  ranging from 1 to  $\min(k, t)$ ,

if  $u > 0$ , are such that  $zz$  is equal to the product modulo  $p_j$  of  $za$  by each of the  $2^k$   $2^k$ -th roots of unity, where  $za$  is the value obtained for  $w$  according to claim 15, and

for each such number  $zz$ , a value for the component  $Q_{i,j}$  is obtained by taking  $Q_{i,j}$  equal to  $zz$  if the equation  $G_i \equiv Q_i^v \bmod n$  is used, or to the inverse of  $zz$  modulo  $p_j$  if  $G_i \cdot Q_i^v \equiv 1 \bmod n$  is used for this value of  $i$ .

21. (New) A computer-implemented process for producing asymmetric cryptographic keys, said keys comprising  $m \geq 1$  private values  $Q_1, Q_2, \dots, Q_m$  and  $m$  respective public values  $G_1, G_2, \dots, G_m$ , the computer-implemented process comprising:

selecting a security parameter  $k$ , wherein  $k$  is an integer greater than 1;

selecting  $m$  base numbers  $g_1, g_2, \dots, g_m$ , wherein each base number  $g_i$  (for  $i = 1, \dots, m$ ) is an integer greater than 1;

determining a modulus  $n$ , wherein  $n$  is a public integer equal to the product of at least two prime factors  $p_1, \dots, p_f$ , at least two of these prime factors, say  $p_1$  and  $p_2$ , being such that  $p_1 \equiv 3 \bmod 4$ ,  $p_2 \equiv 3 \bmod 4$ , and such that  $p_2$  is complementary to  $p_1$  with respect to one of the base numbers;

calculating the public values  $G_i$  for  $i = 1, \dots, m$  through  $G_i \equiv g_i^2 \pmod{n}$ ; and  
 calculating the private values  $Q_i$  for  $i = 1, \dots, m$  by solving either the equation  
 $G_i \cdot Q_i^v \equiv 1 \pmod{n}$  or the equation  $G_i \equiv Q_i^v \pmod{n}$ , wherein the public exponent  $v$  is such that  
 $v = 2^k$ .

22. (New) The computer-implemented process according to claim 21, wherein the number  
 $(f - e)$  (where  $e \geq 0$ ) of prime factors of the modulus  $n$  which are congruent to 3 mod 4 is  
 larger than 2, and those prime factors  $p_{j+1}$  for  $2 \leq j \leq m$  which are congruent to 3 mod 4 are  
 determined iteratively as follows:

the profile  $\text{profile}_j(g_j)$  of  $g_j$  with respect to the prime factors  $p_1, p_2, \dots, p_j$  is computed,  
 and

if  $\text{profile}_j(g_j)$  is flat, then the prime factor  $p_{j+1}$  is chosen such that  $p_{j+1}$  is  
 complementary to  $p_1$  with respect to  $g_j$ ; else, a number  $g$  is chosen among the  $(j - 1)$  base  
 numbers  $g_1, g_2, \dots, g_{j-1}$  and all of their multiplicative combinations, such that  
 $\text{profile}_j(g) = \text{profile}_j(g_j)$ , then  $p_{j+1}$  is chosen such that  $\text{profile}_{j+1}(g_j) \neq \text{profile}_{j+1}(g)$ ,

wherein the last prime factor  $p_{f-e}$  congruent to 3 mod 4 is, in the case that  $f - e \leq m$ , chosen  
 such that  $p_{f-e}$  is complementary to  $p_1$  with respect to all of the base numbers  $g_i$  such that  
 $f - e \leq i \leq m$  and whose profile  $\text{profile}_{f-e-1}(g_i)$  is flat.

23. (New) The computer-implemented process according to claim 21, wherein the number  $e$   
 of prime factors of the modulus  $n$  which are congruent to 1 mod 4 is at least equal to 1, and each  
 such prime factor is determined as follows:

a candidate prime number  $p$  is chosen, such that the Legendre symbol of each base number  $g_i$  (for  $i = 1, \dots, m$ ) with respect to  $p$  is equal to  $+1$ ,

the integer  $t$  is computed such that  $(p-1)$  is divisible by  $2^t$ , but not by  $2^{t+1}$ ,

the integer  $s = (p-1+2^t)/2^{t+1}$  is computed,

an integer  $b \equiv h^{p-1/2^t} \pmod{p}$ , where  $h$  is a non-quadratic residue of the body of integers modulo  $p$ , is computed,

the  $m$  integers  $r_i \equiv g_i^{2^s} \pmod{p}$  for  $i = 1, \dots, m$  are computed,

an integer  $u$  is initialized to  $u = 0$ ,

the following sequence of steps, where  $i$  is initialized to 1, is iteratively implemented:

an integer  $w$  is initialized to  $w = r_i$ ,

if  $r_i = \pm g_i$ , the value of  $i$  is incremented and a sequence of steps with the new value of  $i$  is proceeded to if  $i < m$ , whereas the candidate prime number  $p$  is accepted as a factor of the modulus  $n$  if  $i = m$ ,

if  $r_i \neq \pm g_i$ :

an integer  $jj$  is initialized to 1,

the following sequence of steps, where an integer  $ii$  is initialized to 1, is iteratively implemented:

$x \equiv w^2 / g_i^2 \pmod{p}$  is computed,

$y \equiv x^{2^{t-ii-1}} \bmod p$  is computed, and

if  $y = +1$ , the sequence is terminated at the current value of  $ii$ ,

if  $y = -1$ ,  $jj$  is assigned the value  $jj = 2^{ii}$ , the number  $w$  is assigned a new value equal to the old value multiplied by  $b^{jj}$  modulo  $p$ , and

for  $ii < t - 2$ , the value of  $ii$  is incremented and a new iteration is proceeded to with the new value of  $ii$ ,

for  $ii = t - 2$ , the value of number  $u$  is updated through the relation  $jj = 2^{t-u}$ , and

if  $t - u < k$ , the candidate prime number  $p$  is rejected as a factor of the modulus  $n$ ,

if  $t - u > k$ , the value of  $i$  is incremented and a sequence of steps with the new value of  $i$  is proceeded to if  $i < m$ , whereas the candidate prime number  $p$  is accepted as a factor of the modulus  $n$  if  $i = m$ .

24. (New) The computer-implemented process according to claim 21, wherein, to compute the  $f \cdot m$  private components  $Q_{i,j}$  of the private values  $Q_1, Q_2, \dots, Q_m$ , the following steps are implemented for each couple  $(i, j)$ :

an integer  $t$  is determined, which is equal to 1 if  $p_j$  is congruent to 3 mod 4, and to the value obtained for  $t$  according to claim 15 if  $p_j$  is congruent to 1 mod 4,

an integer  $u$  is determined, which is equal to 0 if  $p_j$  is congruent to 3 mod 4, and to the value obtained for  $u$  according to claim 15 if  $p_j$  is congruent to 1 mod 4,

the integer  $z \equiv G_i^s \pmod{p_j}$  is computed, where  $s = (p - 1 + 2^t) / 2^{t+1}$ ,

all the numbers  $zz$  are being considered, which:

if  $u = 0$ , are such that  $zz = z$  or such that  $zz$  is equal to the product modulo  $p_j$  of  $z$  by each of the  $2^{i-1}$   $2^i$ -th primitive roots of unity, for  $i$  ranging from 1 to  $\min(k, t)$ ,

if  $u > 0$ , are such that  $zz$  is equal to the product modulo  $p_j$  of  $za$  by each of the  $2^k$   $2^k$ -th roots of unity, where  $za$  is the value obtained for  $w$  according to claim 15, and

for each such number  $zz$ , a value for the component  $Q_{i,j}$  is obtained by taking  $Q_{i,j}$  equal to  $zz$  if the equation  $G_i \equiv Q_i^v \pmod{n}$  is used, or to the inverse of  $zz$  modulo  $p_j$  if  $G_i \cdot Q_i^v \equiv 1 \pmod{n}$  is used for this value of  $i$ .